# Lecture 1A: Introduction, Proposition Logic

UC Berkeley EECS 70
Summer 2022
Tarang Srivastava

# Course Overview

**Course Webpage: www.eecs70.org**

Explains policies, calendar for OH, HW, midterm dates, schedule, etc

**Course Format**

Lecture → Mon-Thu 12:30-2p Dwinelle 155 (and live Zoom/recorded)

Discussion → Mon-Thu. Will cover content from that day's lecture.

Office Hours → See eecs70.org/calendar for location and times. Submit tickets on oh.eecs70.org

# Course Overview (cont.)

**Software** "Media Gallery"

bCourses → Lecture recordings

Gradescope → HWs and Vitamins

Piazza → Questions, Communications, Everything else!

Email: cs70-staff@berkeley.edu → Personal questions, extenuating circumstances, etc

Top Bar Attendance Form → Attendance Credit

**Weekly Post**

On Piazza. It is required reading every week.

# Course Overview (cont.)

**Check you are enrolled in these services**

bCourses, Piazza, Gradescope. Please email cs70-staff@berkeley.edu if not enrolled.

**DSP**

You should have received an email from Nikki Suzani. Please email us if you have not.

**Incomplete**

If you are finishing an incomplete this semester please email us with the conditions of your incomplete.

# Assignments

**Homework** → released weekly on Saturday morning

We will drop lowest two scores

Due every Thursday. No penalty grace period until Friday 11:59 pm. Graded on accuracy.

Material from last WTh and this MTue

**Vitamins** → released weekly on Saturday morning

Due every Thursday. No penalty grace period until Friday 11:59 pm Graded on accuracy. Instant feedback on your answers.

Material from this week's MTuWTh lecture

**Discussion Attendance**

1 point for each discussion. 13 needed for full credit

**Exams**

Midterm 7/15 Time 6-8p, Final 8/12 Time 6-9p.

No alternates

| | |
|---|---|
| **Discussion Attendance** | 5% |
| **Vitamin** | 5% |
| **Homework** | 20% |
| **Midterm** | 30% |
| **Final** | 40% |

# Website

https://www.eecs70.org/

Zoom link

OH Times

TAs email

how keep your spot

M OH

CS 70 | Lecture | Discussions | Calendar | Policies | Resources | Staff | Attendance | Piazza | Queue

Syllabus — most read!

Google Form

TA will give you a secret word

## Discrete Mathematics and Probability Theory

CS70 at UC Berkeley, Summer 2022

Jingjia Chen, Michael Psenka, and Tarang Srivastava

Lecture: MTuWTh 12:30 pm - 1:59 pm, Dwinelle 155

**Jump to current week**

Slides

read before lecture

| Week | Date | Lecture | Resources | Notes | Discussion | Homework |
|---|---|---|---|---|---|---|
| 1 | Mon 6/20 | *Juneteenth* | | | Disc 1A, solutions | |
| | Tue 6/21 | Introduction, Propositional Logic | | Note 0<br>Note 1 | Disc 1B, solutions | HW 1, solutions |
| | Wed 6/22 | Proofs | | Note 2 | Disc 1C, solutions | |
| | Thu 6/23 | Induction | | Note 3 | | |

recording

EOD

release on SAT

# Instructors

Tarang: First third of the course

Michael: Secord third of the course

Jingjia: Last third of the course

Tarang Srivastava (he/him)

tarang.sriv@ • website

Hi! I'm a fourth year Math and CS double major. I have been a TA for 5 semesters and Head TA for 3, I'm very excited to be teaching yall this semester!

Michael Psenka (he/him)

psenka@ • website

I'm a 2nd year PhD student in BAIR–I currently work on representation learning in computer vision and robotics. I did my undergrad in math, and I continue to enjoy bringing my math nerdiness into my CS research. Outside of work, I play piano (& attempt at music production), Smash, chess, and snowboard.

Jingjia Chen (she/her)

jingjia.chen@

# Collaboration

We highly encourage collaboration! So, let's define what that means. (Professor Sinclair)

Discussing approaches to problems is encouraged!

As long as you reach a good understanding of the final solution

You should not allow concerns for cheating to get in the way of discussing problems with your peers

How we recommend collaborating…

- Post on Piazza and read the relevant homework threads
- Come to OH. It's okay to just chill there even if you have no questions

Cases of Academic Misconduct will be dealt with by the course staff and Center for Student Conduct

# Why CS70?

Programming + Microprocessors → Superpower

What are your computers doing?

Logic and Proofs!

Ex: Induction = Recursion

What can computers do?

Work with discrete objects

Discrete Math → immense applications

Computers learn and interact with the world?

Probability → Ex: machine learning, data analysis, robotics,

Our goal: ***teach you to think more critically and powerfully…and to deal clearly with uncertainty itself.***

# Tips for CS70

**READ THE NOTES! READ THE NOTES! READ THE NOTES!**

- Reading mathematical text is not the same as reading regular non-fiction.
- Read non-linearly. Jump around. Keep a pencil in hand. Work out examples.
- We will hold specific OH this week to give some tips on how to best read the notes. This is a skill we hope you pickup in this class. Thu
- Reading the notes takes time. Allocate 1-2 hours for each note
- There is a myth that you need “mathematical maturity” to do well in this course.
- Give yourself plenty of time to think about homework problems.

# Announcements!

- Join Piazza. Read the Welcome Post
- Discussions start today, signup link is on Piazza Do it today
- Office Hours start today, see course calendar on website
- **HW 1** and **Vitamin 1** have been released, due Thu (grace period Friday)

# Propositions: Statements that are true or false

| **Statement** | **Is it a proposition?** | **true/false?** |
|---|---|---|
| Square root of 2 is irrational | Yes, proposition | true |
| 2 + 2 = 4 | Yes, prop. | true |
| 2 + 2 = 3 | Yes, prop. | false |
| Tom Hanks is in Forrest Gump | Yes, prop | true |
| Tom Hanks is a good actor<br>fuzzy words | No its not prop | – |
| 2 + 2 | No | – |
| 2 + x = 5 Free variable | No | – |
| Any even > 2 is a sum of 2 primes | Yes, prop | False |

# Using variables to denote propositions

P = "I am Oski"    Q = "I am Carol Christ"

| Operation | Symbol | Meaning | Example |
|---|---|---|---|
| Conjunction | $\boxed{P \land Q}$ | P AND Q must both be true | I am oski and I am carol christ |
| Disjunction | $P \lor Q$ | P OR Q is true | I am oski or I am carol christ |
| Negation | $\neg P$ | not P | I am not oski |

# Truth Tables

A way to systematically record what an operation on propositions is doing.

| $P$ | $Q$ | AND<br>$P \wedge Q$ | OR<br>$P \vee Q$ | $\neg P \vee Q$ | $\neg P$ | $P \vee \neg P$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | F | T |
| T | F | F | T | F | F | T |
| F | T | F | T | T | T | T |
| F | F | F | F | T | T | T |

Law of the excluded middle: $P$ is true or $\neg P$ is true (but not both)

A proposition that is always true tautology $(P \vee \neg P)$

A proposition that is always false contradiction $(P \wedge \neg P)$

# Implications

if you stand in the rain, then you will get wet

P ⟹ Q

If P, then Q

P implies Q

P = "You like CS70"

Q = "You like probability"

| P | Q | $P \Rightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

vacuously true

- if you like 70, then you like prob.
- You like CS70 only if you like prob.
- The fact that you like probability is a necessary condition for you to like CS70

if pigs can fly, then everyone in 70 will get an A

# Converse, Inverse and Contrapositive

| $P$ | $Q$ | $\neg P$ | $\neg Q$ | $P \Rightarrow Q$ | Converse $Q \Rightarrow P$ | Inverse $\neg P \Rightarrow \neg Q$ | Contrapositive $\neg Q \Rightarrow \neg P$ | $P \Leftrightarrow Q$ |
|---|---|---|---|---|---|---|---|---|
| T | T | F | F | T | T | T | T | T |
| T | F | F | T | F | T | T | F | F |
| F | T | T | F | T | F | F | T | F |
| F | F | T | T | T | T | T | T | T |

Converse: if you like prof., then you like 70

Inverse:

Contrapositive

$P \Rightarrow Q \wedge Q \Rightarrow P \equiv P \Leftrightarrow Q$

P if and only if Q

P iff Q

# Logical Equivalence

**Propositional formula** is an expression made up of propositional variables combined with logical operators.

Two propositional formulas are **logically equivalent** if they have the same truth table.

Example:

| $P$ | $Q$ | $P \Rightarrow Q$ | $\neg Q \Rightarrow \neg P$ | $\neg P \vee Q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

Contraposition is logically equivalent to the implication

$$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P \equiv \neg P \vee Q$$

# DeMorgan's Law

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

| $P$ | $Q$ | $P \wedge Q$ | $\neg(P \wedge Q)$ | $\equiv$ $\neg P \vee \neg Q$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | T | T |

1) $P \wedge (Q \vee R) \equiv ?$

2) $P \wedge (Q \wedge R) \equiv$

3) $(P \wedge Q) \vee (R \wedge S) \equiv ?$

# Predicates and Quantifiers

$Q(x)$

quantifier

for all $x \in \mathbb{N}$ universe

$\forall$ $(2x$ is even$)$

$(\forall x \in \mathbb{N})(2x$ is even$)$

Predicates: Statements with free variables. Ex: Q(x) = "2x is even"

Predicates by themselves are **not** propositions. Adding a quantifier and a universe allows us to state multiple propositions at once.

Example:

for all natural numbers $n$, $n^2 + n + 41$ is prime

quantifier universe

From Note 0:

$\mathbb{N} = 0, 1, 2, 3 \ldots$

$\mathbb{Z} = \ldots -2, -1, 0, 1, \ldots$

$\mathbb{Z}^+ = 1, 2, 3, 4 \ldots$

$\mathbb{Q}$ : $p/q$ for $p, q \in \mathbb{Z}$

$\mathbb{R}$ : real numbers

$S = \{ ♡, \triangle, \square \}$

$(\forall n \in \mathbb{N})(n^2 + n + 41$ is prime$)$

$0^2 + 0 + 41$ is prime

$1^2 + 1 + 41$ is prime

$2^2 + 2 + 41$ is prime

$\vdots$

# "For All" and "Exists"

$\forall$ "For all" means for all the values in the universe P(x) is true

$\exists$ "Exists" means there is at least one value x in the universe for which P(x) is true

Practice "Every nonzero rational number can be multiplied by some rational number to get 1"

"set minus"

$$\left[ (\forall q \in \mathbb{Q} \setminus \{0\})(\exists x \in \mathbb{Q})(q \cdot x = 1) \right]$$

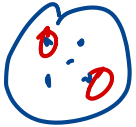
$\forall q \in \mathbb{Q}$ where $q \neq 0$

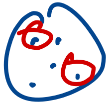
# Logical Equivalence with Quantifiers

$P(x,y$

$\mathbb{N}$

$\exists x \exists y \; P(x,y) \equiv \exists y \exists x \; P(x,y)$





$3x+5 > y$

the number multiples of $x$ are 7 more than $y$ h

totient function

$\forall x \exists y \; P(x,y) \not\equiv \exists x \forall y \; P(x,y)$

true is always a bigger number

such a $x$ that all the numbers are greater than it

$y > x$

$P(x,y) = y > x$

Universe $= \mathbb{Z}$

# DeMorgan's Law for Quantifiers

$$\neg(\forall x \in S)\, P(x) \equiv (\exists x \in S)\, \neg P(x)$$

Example:

$P(x)$ $\quad x^2 > 10$

$S = \{1, 2, 3, 4\}$